A Numerical Investigation of the Ising Model

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INTRODUCTION

- Describes a lattice with a number of spins
- Each box represents a spin that can be in 1 of 2 states: black or white (or for the three color one, 1 of 3 states)
- Spin flips or doesn't flip based on whether it will lower or raise energy level
- The critical temperature distinguishes between order (1 color dominates) and disorder (no colors dominate)

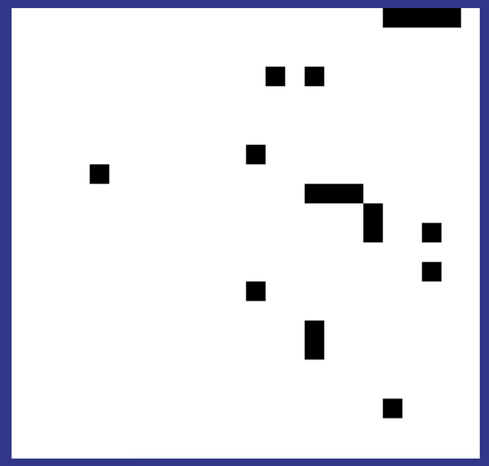
OBJECTIVES

- Explore the relationship between critical temperature and magnetization
- Model phase transitions between states as the temperature changes:
  - Liquid → gas
  - Magnetic → not magnetic

METHODS

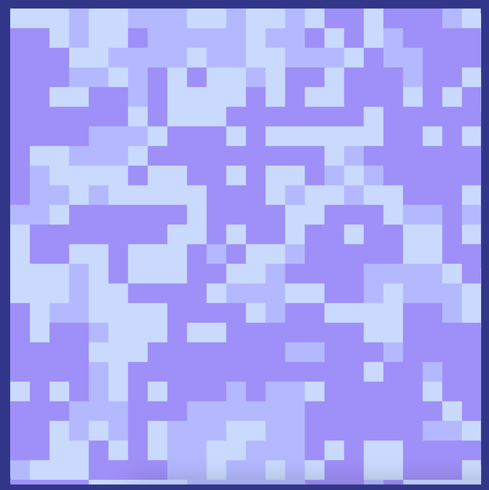
- Calculate energy of configuration by summing values of nearest-neighbor relations for each spin.
- Flip the spins based if it lowers the energy
  - If flipping does lower the energy → accept the flip
  - If not → accept the flip with a low probability (modeled by $r < e^{-\Delta E/T}$, where r is a random number between 0 and 1).
- Calculate magnetization by taking $\sum_{i,j} S_{i,j}$, the absolute value of (number of black squares) - (number of white squares)
- 3 colors: flips to whichever color lowers energy the most, if not, attempt to flip whichever color increases the least

LOW TEMP (Ferromagnet) 2 COLORS



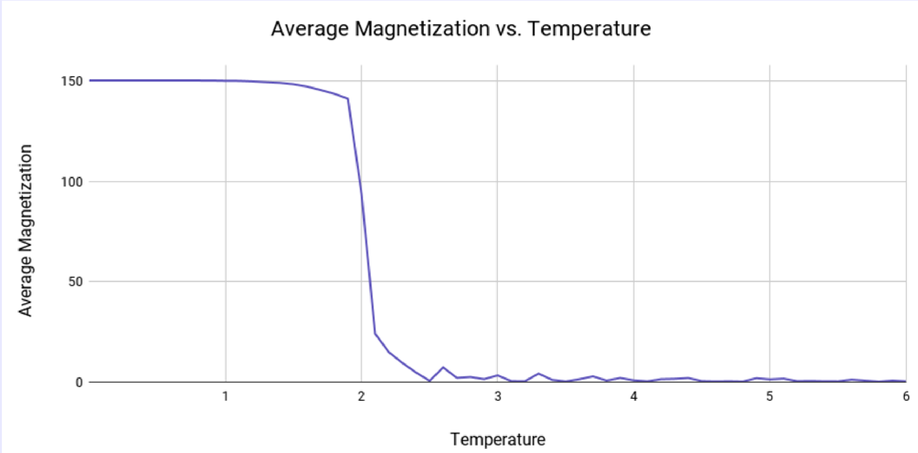
HIGH TEMP (Paramagnet) 2 COLORS

HIGH TEMP (Paramagnet) 3 COLORS



RESULTS

- Average Magnetization vs. Temperature graph can be used to find the critical temperature
- Magnetization at low temperatures is very high, at high temperatures it is very low (close to 0)



FUTURE DIRECTIONS

- Exploring magnetization and critical temperature in a three-color model
- Effects of increasing the number of colors on the critical temperature

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