# Analysis and Applications of Grover's Algorithm

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## Abstract

Imagine a bookshelf with N books in arbitrary order. Any classical algorithm will need to look at an average of N/2 positions in order to find the book with an efficiency of O(N). Quantum algorithms can utilize a superposition of states and simultaneously examine multiple positions. With proper adjustment of phases, the desired book can be found within O(√N) operations.[1]

## Applications

Grover's Algorithm can provide better efficiency on many problems.

**Collision Problem:** Finding collisions in a function(ex: hash functions)

- Classical algorithms for finding this typically have a efficiency of O(N). In contrast, Grover's algorithm provides better efficiency with O(√N)

**Sudoku Problem:** an NP-complete problem

- Finding a solution can be computationally very difficult for classical algorithms as the size increases

**Optimization Problem:** Finding the most efficient choice/pathway

- Problems such as the travelling salesman problem are very complex(O(N!) for a naive solution). Grover's brings this down to O(√N!), but is still slower than the dynamic programming approach.[2]

## Controversies and classical advantage

Theoretical quantum advantage is clearly seen as Grover's offers a quadratic speedup when using iterations as the metric for efficiency.

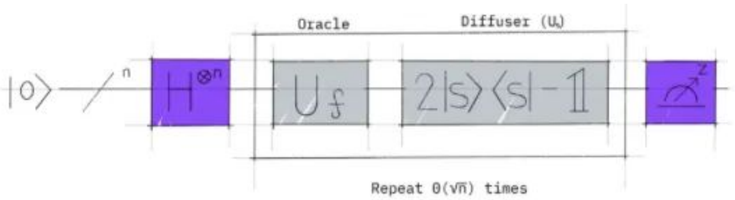
However, there have been a recent controversy concerning whether Grover's provides an advantage to classical systems.

Stoudenmire & Waintal[3] argue that Grover's provides no quantum advantage over a classical implementation. However, their argument about Grover's unreliability stems from their omission of quantum error correction and faulty logic regarding the nature of the advantage it offers, thus largely invalidating their claim.

In practice, Grover's Algorithm is primarily limited by quantum noise, complex oracle creation, and its probabilistic nature.

## How it works

Grover's Algorithm assumes that the number of solutions (N), is a power of 2, such that $N = 2^n$ where n is a positive integer




**1) Preparing the System**

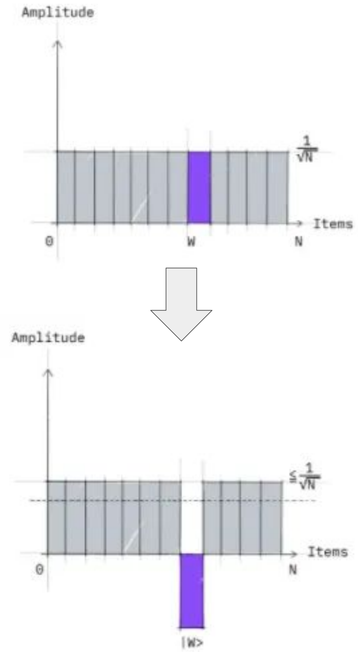
Grover's initializes $2^n$ qubits to |0> and applies a Hadamard gate to create an equal superposition of all states |s>

**2) Amplitude Amplification**

Oracle - passes all states |s> through a black box which changes the phase of the value(s) |W> we are looking for to a negative phase

Diffuser - reflects |s> over the average amplitude, increasing the probability of returning |W>

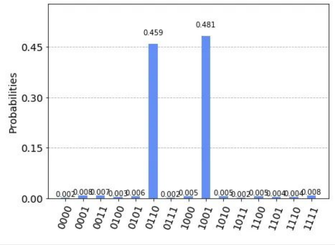
By repeating these 2 steps proportional to √N times, the probability of returning |W> becomes reasonably close to 100%.




**Application:**

- In a 2x2 sudoku grid with possible values of 0 or 1, each row or column cannot repeat values
- The oracle and diffuser can be applied many times to increase the accuracy
- The results of simulating this will most often be 0110 and 1001, the two solutions to the problem

| | |
|---|---|
| $V_0$ | $V_1$ |
| $V_2$ | $V_3$ |




## Citations

[1]Grover, L. K. (1996, July). A fast quantum mechanical algorithm for database search. In *Proceedings of the twenty-eighth annual ACM symposium on Theory of computing* (pp. 212-219).

[2]Bergamaschi, T. R.-W. (2020, February 2). *Quantum Approximate Optimization Algorithms on the "Traveling Salesman Problem."* MIT 6.S089—Intro to Quantum Computing. https://medium.com/mit-6-s089-intro-to-quantum-computing/quantum-approximate-optimization-algorithms-on-the-traveling-salesman-problem-703b8aee6624

[3]Stoudenmire, E. M., & Waintal, X. (2023). Grover's Algorithm Offers No Quantum Advantage. *arXiv preprint arXiv:2303.11317*.

**See also:** https://scottaaronson.blog/?p=7143

**Images:** Parkinson, A. (2020, October 12). *Solving Sudoku Using Quantum Computing*. Medium. https://averyparkinson23.medium.com/solving-sudoku-using-quantum-computing-cbc8a397a504