CHSH Game: Quantum Probability

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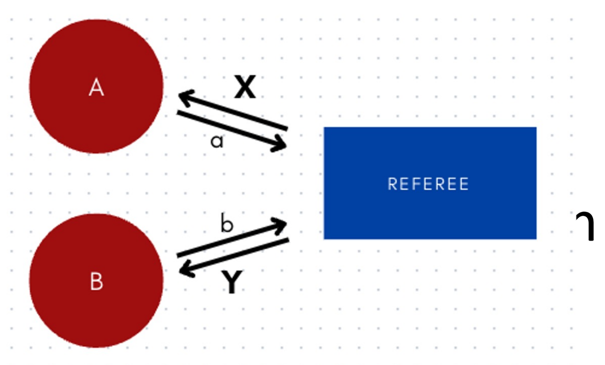
Introduction:

The CHSH Game is a quantum model representing how quantum mechanics can be used to gain advantages that are not possible in the classical world. In the game, there are two players, Alice and Bob. Alice and Bob are not able to communicate during the game but can communicate before the game starts.

The game works as follows:

1. A referee chooses X, Y ∈ {0, 1} uniformly at random.
2. The referee gives Alice X and Bob Y.
3. Alice responds with a ∈ {0, 1} and b ∈ {0, 1}.



If both Alice and Bob are given 1 (X=Y=1), they win when they return different outputs (ex: a=0, b=1). Otherwise (ex: X=1, Y=0), they win when they output the same response (ex: a=1, b=1).

Thus, in the classical sense, their best option for winning is to choose one number to always return before the game starts (ex: always a=0, b=0), giving them a 75% chance at success. However, using quantum mechanics, the percentage rises.

Results:

Using the computational model of the CHSH game, we can see that the win percentage in the quantum game (85%) is **around 10% higher** than the win percentage in the classical game (75%).

The diagram shows how Alice and Bob have a higher chance of winning with the entangled qubits than the classical version of the game due to the quantum definition of particles. Given that Alice and Bob were able to predetermine the state of their qubits, the angle measure of θ has allowed for the different probabilities of measuring and thus sending back outputs (0 or 1).

Conclusion:

The CHSH Game is a dual sided model of Bell's Theorem and entanglement. Bell's Theorem contradicts local hidden-variable theories, which suggests a change or information acting on one particle will not affect the other.

The quantum side of the game relies both on the correlation of qubit states as well as the knowledge that once measured, the original qubit state shall change to the basis measured upon.

By furthering our understanding of entanglement, we can show how quantum machines can accomplish tasks more efficiently compared to classical machines.

Visuals:

Classical:

```python
# CHSH game simulation

game_rounds = 100000
wins = 0
losses = 0

for i in range(game_rounds):
    # referee gives Alice and Bob one random bit each
    alice_ref = np.random.randint(2)
    bob_ref = np.random.randint(2)

    alice_bit = generate_bit_classical(alice_ref)
    bob_bit = generate_bit_classical(bob_ref)

    # Win conditions
    if alice_ref==bob_ref==1:
      if alice_bit != bob_bit:
        wins += 1
      else:
        losses += 1
    elif (alice_ref==1 and bob_ref==0) or (alice_ref==0 and bob_ref==1) or (alice_ref==0 and bob_ref==0):
      if alice_bit == bob_bit:
        wins += 1
      else:
        losses += 1

win_percentage = wins/game_rounds*100
print("Wins: {}%".format(win_percentage))
```

```
Wins: 74.913%
```

Quantum:

```python
# CHSH game simulation

game_rounds = 1000
wins = 0
losses = 0

for i in range(game_rounds):
    # First, Alice and Bob create a Bell pair and take one qubit each
    bell_pair = create_bell_pair(0)

    # referee gives Alice and Bob one random bit each
    alice_ref = np.random.randint(2)
    bob_ref = np.random.randint(2)

    # Alice and Bob each generate their return bit by measuring their half of the bell pair
    alice_bit = generate_alice_bit_quantum(alice_ref, bell_pair)
    bob_bit = generate_bob_bit_quantum(bob_ref, bell_pair)

    # Win conditions
    if alice_ref==bob_ref==1:
      if alice_bit != bob_bit:
        wins += 1
      else:
        losses += 1
    elif (alice_ref==1 and bob_ref==0) or (alice_ref==0 and bob_ref==1) or (alice_ref==0 and bob_ref==0):
      if alice_bit == bob_bit:
        wins += 1
      else:
        losses += 1

win_percentage = wins/game_rounds*100
print("Wins: {}%".format(win_percentage))
```

```
Wins: 85.0%
```

Alice and Bob predetermine the Bell state of the entangled qubits such that the spins are perfectly correlated and pick 2 different measurement bases perpendicular to each other.

Original Qubit State: $\frac{|00\rangle + |11\rangle}{\sqrt{2}}$

If Alice:

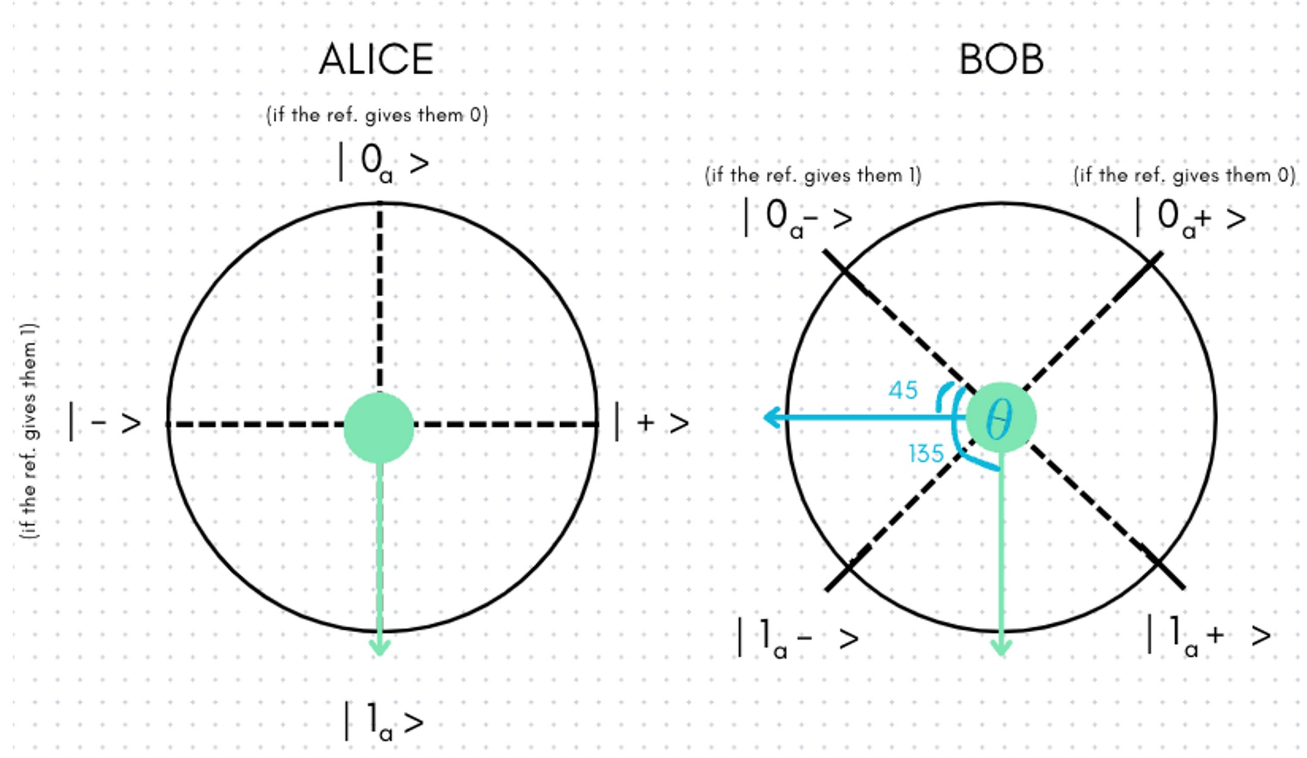
Is given a **0** by the ref (**$|0_a, 1_a\rangle$), she will get a 50-50%** chance of measuring a 0 or a 1 to send back.

Is given a **1** by the ref (**|-, + >**), she will have a 50-50% chance measure a **|+> or a |->** and will send back a **0 or a 1 respectively.**

If Bob:

Is given a **0** by the ref (**|0-, 1+ >),** he will use the given formula and the measure of angle θ to determine the probability of sending back a 0 or a 1. This is the same if Bob is given a **1** (**|1-, 0+ >**).

θ = Angle between spin of particle and basis. Specific focus on θ=45°, θ=135°



$\cos^2(\theta/2)$ = Probability of measuring a 1

$\cos^2(45°/2) \cong 0.854$ or (85.4%)

$\cos^2(135°/2) \cong 0.146$ or (14.6%)

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