Section 5: Drift and Diffusion

Saturday, October 16, 2021 7:44 PM

I. Carrier Concentration Numerical Example

For Silicon,

$$N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$$
$$N_v = 1.04 \times 10^{19} \text{ cm}^{-3}$$
$$E_g = 1.12 \text{ eV}$$

a) What is n_i as a function of temperature? At $T = 300\text{K}$?

b) How does this compare to the atomic density of Silicon?

$$a = 5.43 \text{ Å}$$

c) What is n, p if $N_D = 10^{17} \text{ cm}^{-3}$? Where is E_f? (Assume $T = 300\text{K}$). Take $n_i = 10^{10} \text{ cm}^{-3}$.

d) Suppose $E_f - E_i = -0.3 \text{ eV}$. Is the material n or p type? What is n and p?

a) $n_i^2 = N_c N_v e^{-E_g/k_B T}$

Plugging in the values,

$$n_i^2 = 2.912 \times 10^{38} \times e^{-1.12/k_BT}$$

$$\Rightarrow \boxed{n_i = 1.706 \times 10^{19} \times e^{-1.12\,eV/k_BT}}$$

With $T = 300\,K$, take $k_BT = 25.9$ meV

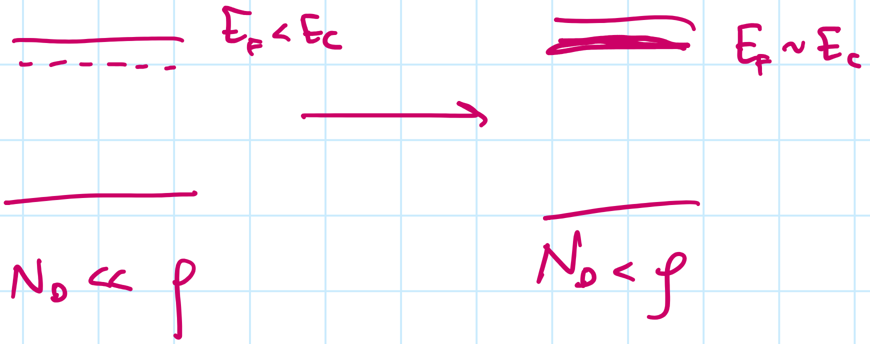
$$\Rightarrow \boxed{n_i \simeq 6.95 \times 10^{9} \ \text{cm}^{-3}}$$

Interestingly, this does not match the oft quoted value of $n_i = 10^{10}\ \text{cm}^{-3}$ or $n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$, even in the same textbooks where the values of N_c and N_v I quoted appear. This is an inconsistency which appears to be a result of experimental values of n_i clashing with theoretical calculations of N_c, N_v since Si is not actually that simple of a material. For the purpose of this class though, just take $n_i = 10^{10}\ \text{cm}^{-3}$.

b) We can estimate the atomic density of Si to be on the order of of Si to be on the order of

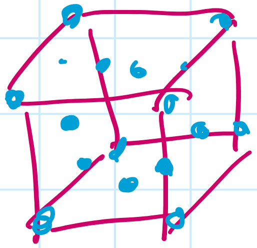
$$\rho \sim \frac{1}{a^3} = 6.25 \times 10^{21}\ \mathrm{cm}^{-3}$$

Clearly, the intrinsic carrier concentration is basically nothing compared to the total atomic density. It's interesting to consider what happens when the doping level approaches this value — at what point is the material still itself? For Silicon, it is generally accepted that $>10^{19}\ \mathrm{cm}^{-3}$ is considered "degenerate doping" and $\sim 10^{20}$ is a realistic upper limit to the doping amount. At this point the defect levels essentially form a continuous band and the Fermi level is roughly at the band edges (usual equations no longer valid).



Actually, because Si is an "fcc" lattice,

ρ is slightly different



8 corner atoms — each shared w/ 8 cells

6 face-centered atoms — each shared w/ 2 cells

4 internal atoms — each shared w/ 1 cell

$$\Longrightarrow \quad 8 \times \frac{1}{8} + 6 \times \frac{1}{2} + 4 = 8 \text{ atoms/cell}$$

$$\therefore \quad \rho = \frac{8 \text{ atoms}}{a^3} \approx 5 \times 10^{22} \text{ cm}^{-3}$$

c) Room T : good assumption everything is ionized

$$n = N_D - \cancel{N_A} = N_D = 10^{17} \text{ cm}^{-3}$$

$$p = n_i^2 / n = 10^{20} / 10^{17} = 10^3 \text{ cm}^{-3}$$

$$n = N_c e^{-(E_c - E_F)/k_B T}$$

$$\Longrightarrow \quad E_c - E_F = k_B T \ln \frac{N_c}{n}$$

$$E_F \approx E_c - k_B T \ln \frac{N_c}{n}$$

2.8×10^{19}

$$E_F \approx E_c - k_B T \ln \frac{}{n}$$

$$= E_c - 25.9 \text{ meV} \times \ln \frac{2.8 \times 10^{19}}{10^{17}}$$

$$\simeq E_c - 0.146 \text{ eV}$$

$$\boxed{\begin{aligned} \therefore \quad n &= 10^{17} \text{ cm}^{-3} \\ p &= 10^{3} \text{ cm}^{-3} \\ E_F &= E_c - 0.146 \text{ eV} \end{aligned}}$$

Alternatively, we can calculate E_F relative to E_i.

$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2k_BT}$$

$$= N_c e^{-(E_c - E_i)/k_BT}$$

$$= N_v e^{-(E_i - E_v)/k_BT}$$

$$n = N_c e^{-(E_c - E_f)/k_BT}$$

$$= N_c e^{-(E_c - E_i + E_i - E_f)/k_BT}$$

$$= N_c e^{-(E_c - E_i)/k_BT} e^{-(E_i - E_f)/k_BT}$$

$$\boxed{\begin{aligned} n &= n_i e^{(E_f - E_i)/k_BT} \\ p &= n_i e^{-(E_f - E_i)/k_BT} \end{aligned}}$$

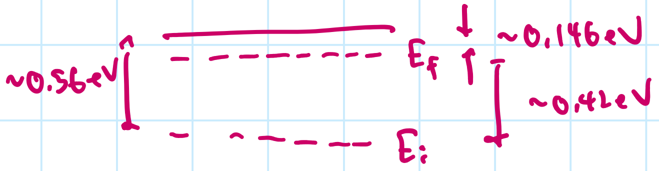
Then, $E_f = k_BT \ln \frac{n}{n_i} + E_i$

$$\simeq k_BT \ln \frac{10^{17}}{10^{10}} + E_i$$

(Note: $k_BT \ln 10 \simeq 60$ meV)

$$\boxed{E_f \simeq E_i + 0.42 \text{ eV}}$$

Does this make sense?



Roughly checks out!

d) $E_f - E_i < 0 \Longrightarrow$ closer to VB

$\boxed{\text{p-type}}$

Use formula from before:

$$n = n_i e^{(E_f - E_i)/k_B T}$$

$$p = n_i e^{-(E_f - E_i)/k_B T}$$

$$\Longrightarrow \boxed{\begin{aligned} n &= 9.3 \times 10^4 \text{ cm}^{-3} \\ p &\simeq 1.07 \times 10^{15} \text{ cm}^{-3} \end{aligned}}$$

⓶ Drift Current

a) Recap of lecture:

$$m \Delta v = \Delta p = F \tau \leftarrow \text{mean time to collision}$$

$$\Longrightarrow \Delta v = \frac{F\tau}{m^*} = \frac{qE\tau}{m^*} = \left(\frac{q\tau}{m}\right) E$$

Δv is the drift velocity v_d

$$\boxed{\begin{array}{ll} v_{d,p} = \mu_p E & v_{d,n} = -\mu_n E \\ \mu_p = q\tau/m_p^* & \mu_n = q\tau/m_n^* \end{array}}$$

To calculate drift current, simply count the number of charges going past a unit area per unit time

charge flowing through surface in $(t, t+\Delta t)$ = charge per carrier × # charges per unit vol × vol in $t, t+\Delta t$

$$\Delta Q_n = -qn\left(A v \Delta t\right)$$

$$\Delta Q_p = qp\left(A v \Delta t\right)$$

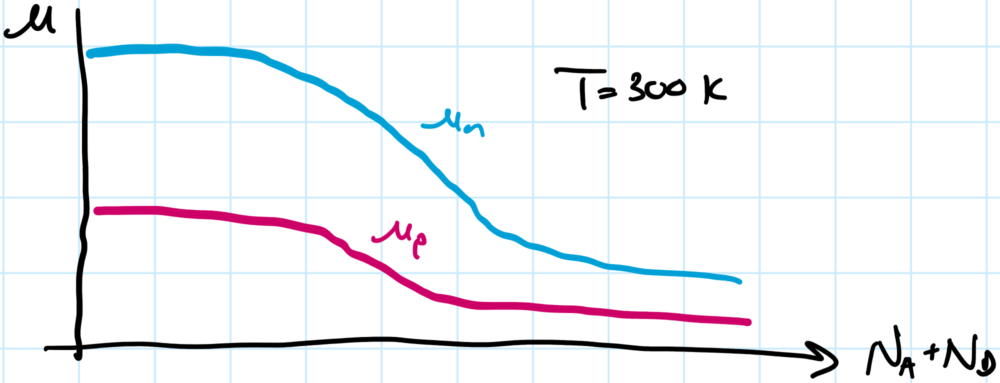
$$j = \Delta Q / A\Delta t = I/A$$

$$\Rightarrow \quad j_n = -qn v_n = +qn\mu_n E$$

$$j_p = qp v_p = qp\mu_p E$$

$$\boxed{\begin{aligned} \dot{j}_{drift} &= \dot{j}_n + \dot{j}_p \\ &= (qn\mu_n + qp\mu_p)E \\ &= \sigma E \\ \text{Ohm's Law} \end{aligned}}$$

b) Mobility vs Dopant Concentration



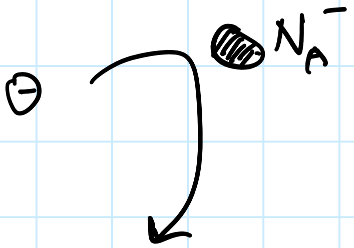
Q: Why is the trend generally decreasing?

A: We can get a hint of what is happening from the x-axis, which is <u>total</u> impurity ion concentration.

Evidently, the phenomenon is due to the presence of dopants, more so than a net n or p-doping.

In fact, the phenomenon is <u>impurity</u>

In fact, the phenomenon is <u>impurity ion scattering</u>: the Coulombic force from the charged impurity ions causes moving charges to change direction, and naturally this gets worse (τ decreases) as more ions are added.



Q: Why is the trend initially flat?

A: This should give a hint that some other effect is dominating in the low doping region. This effect is phonon scattering — lattice vibrations from the finite crystal temperature are the dominant source of scattering in this regime. In fact,

$$\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{ion}}$$

If $\mu_{phonon} \ll \mu_{ion}$, $\mu \propto \mu_{phonon}$

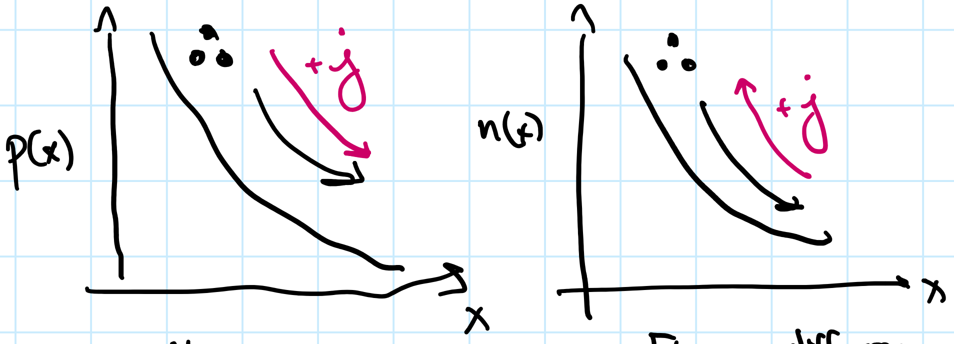
If $\mu_{ion} \ll \mu_{phonon}$, $\mu \propto \mu_{ion}$

Q: Why is the trend flat in the large dopant region?

A: The scattering potential is partially "screened" by the carriers themselves. Essentially, the carriers redistribute to cancel out some of the field.

III. Diffusion Current

a) Hole and electron diffusion



Hole diffusion

Electron diffusion

Expect $j_{p,diff} \propto -\frac{dp}{d}$. Expect $j_{n,diff} \propto \frac{dn}{d}$

Expect $j_{p,\text{diff}} \propto -\frac{dp}{dx}$; Expect $j_{n,\text{diff}} \propto \frac{dn}{dx}$

In fact,

$$\boxed{\begin{aligned} j_{p,\text{diff}} &= -qD_p \frac{dp}{dx} \\ j_{n,\text{diff}} &= qD_n \frac{dn}{dx} \end{aligned}}$$

Generalized version: Fick's first law

$$j = -D \frac{d\rho}{dx}$$

j: diffusion flux (amount of substance per unit area per unit time)

D: diffusivity

ρ: concentration (amount of substance per unit volume)

b) Einstein Relation

In equilibrium, diffusion and drift will balance each other (no net particle flow). Can show this leads to:

$$\boxed{\frac{D_n}{\mu_n} = \frac{k_B T}{q}}$$

(and similarly for holes)

This is a very profound result. A transport coefficient D_n is related to a dissipative process (scattering → μ_n) through thermal fluctuations $k_B T$. This is an example of the

<u>fluctuation - dissipation theorem</u>.

Another example might be drag, whose counterpart is Brownian motion. The Einstein relation would then be:

$$\text{diffusivity} \nearrow \frac{D}{\underset{\text{viscosity}}{\gamma}} = k_B T \leftarrow \text{thermal fluctuations}$$