Section 5: Drift and Diffusion (Worksheet)

Saturday, October 16, 2021 7:44 PM

I. Carrier Concentration Numerical Example

For Silicon,

$$N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$$
$$N_v = 1.04 \times 10^{19} \text{ cm}^{-3}$$
$$E_g = 1.12 \text{ eV}$$

a) What is n_i as a function of temperature? At $T = 300$ K?

b) How does this compare to the atomic density of Silicon?

$$a = 5.43 \text{ Å}$$

c) What is n, p if $N_D = 10^{17}$ cm^{-3}? Where is E_f? (Assume $T = 300$K). Take $n_i = 10^{10}$ cm^{-3}.

d) Suppose $E_f - E_i = -0.3$ eV. Is the material n or p type? What is n and p?

II. Drift Current

a) Recap of lecture:

$$m\Delta v = \Delta p = F\tau$$ ← mean time to collision

$$\Rightarrow \Delta v = \frac{F\tau}{m^*} = \frac{qE\tau}{m^*} = \left(\frac{q\tau}{m}\right)E$$

Δv is the drift velocity v_d

$$\boxed{\begin{array}{ll} v_{d,p} = \mu_p E & v_{d,n} = -\mu_n E \\ \mu_p = q\tau/m_p^* & \mu_n = q\tau/m_n^* \end{array}}$$

To calculate drift current, simply count the number of charges going past a unit area per unit time

charge flowing through surface = charge per carrier × # charges per unit vol × vol in t, t+Δt through surface in $(t, t+\Delta t)$ | carrier | per unit vol | $t, t+\Delta t$

$$\Delta Q_n = -qn\,(A v \Delta t)$$

$$\Delta Q_p = qp\,(A v \Delta t)$$

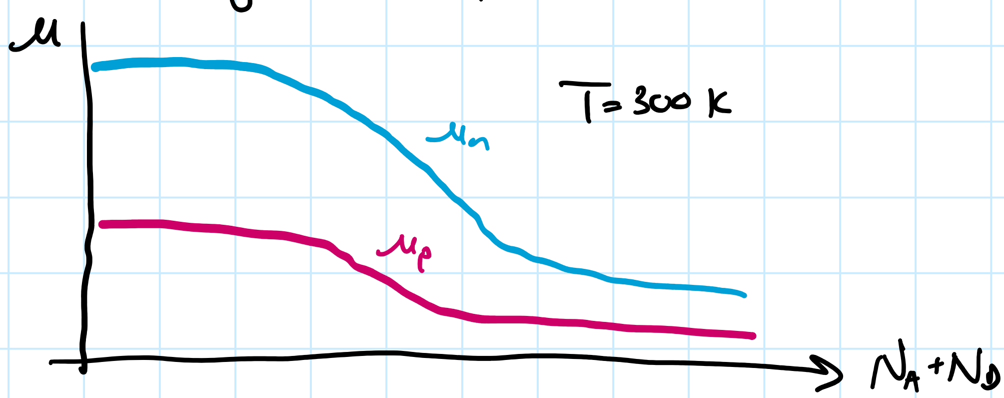
$$j = \Delta Q / A\Delta t \quad = I/A$$

$$\Rightarrow \quad j_n = -qn v_n = +qn\mu_n E$$

$$j_p = qp v_p = qp\mu_p E$$

$$\Rightarrow \quad \boxed{\begin{aligned} j_{drift} &= j_n + j_p \\ &= (qn\mu_n + qp\mu_p)E \\ &= \sigma E \\ &\text{Ohm's Law} \end{aligned}}$$

b) Mobility vs Dopant Concentration



Q: Why is the trend generally decreasing?

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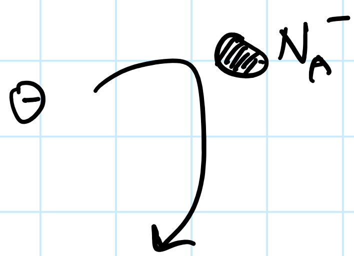
A: We can get a hint of what is happening from the x-axis, which is total impurity ion concentration. Evidently, the phenomenon is due to the presence of dopants, more so than a net n or p-doping.

In fact, the phenomenon is impurity ion scattering: the Coulombic force from the charged impurity ions causes moving charges to change direction, and naturally this gets worse (τ decreases) as more ions are added.



Q: Why is the trend initially flat?

A: This should give a hint that some other effect is

A: This should give a hint that some other effect is dominating in the low doping region. This effect is <u>phonon scattering</u> — lattice vibrations from the finite crystal temperature are the dominant source of scattering in this regime. In fact,

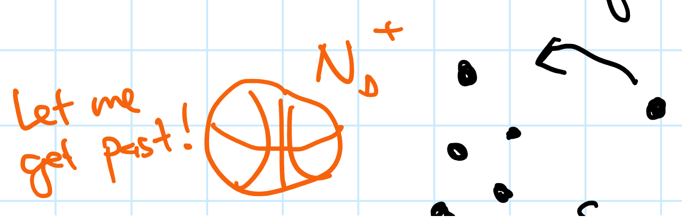
$$\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{ion}}$$

If $\mu_{phonon} \ll \mu_{ion}$, $\mu \approx \mu_{phonon}$

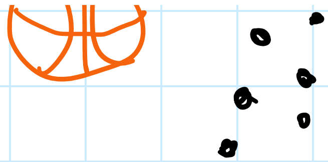
If $\mu_{ion} \ll \mu_{phonon}$, $\mu \approx \mu_{ion}$

Q: Why is the trend flat in the large dopant region?

A: The scattering potential is partially "screened" by the carriers themselves. Essentially, the carriers redistribute to cancel out some of the field.



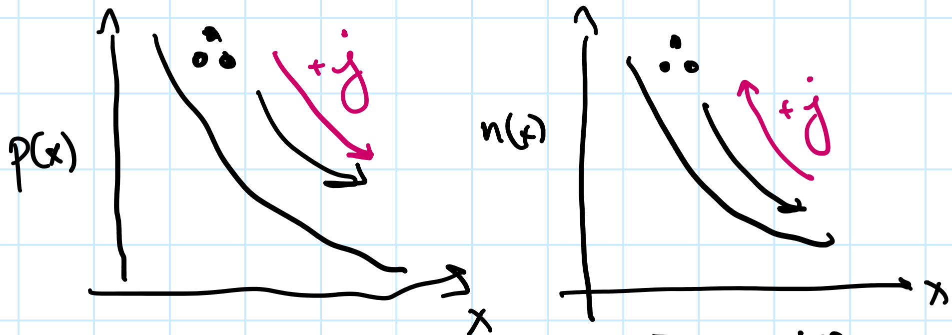
get past!



Screen him!

III. Diffusion Current

a) Hole and electron diffusion



Hole diffusion　　　Electron diffusion

Expect $j_{p,diff} \propto -\frac{dp}{dx}$; Expect $j_{n,diff} \propto \frac{dn}{dx}$

In fact,

$$\boxed{\begin{aligned} j_{p,diff} &= -qD_p \frac{dp}{dx} \\ j_{n,diff} &= qD_n \frac{dn}{dx} \end{aligned}}$$

Generalized version: Fick's first law

$$j = -D \frac{d\rho}{dx}$$

j: diffusion flux (amount of substance per unit area per unit time)

D: diffusivity

[amount of substances

D: diffusivity

ρ: concentration $\left(\begin{array}{c} \text{amount of substance} \\ \text{per unit volume} \end{array} \right)$

b) Einstein Relation

In equilibrium, diffusion and drift will balance each other (no net particle flow). Can show this leads to:

$$\boxed{\frac{D_n}{\mu_n} = \frac{k_B T}{q}}$$

(and similarly for holes)

This is a very profound result. A transport coefficient D_n is related to a dissipative process (scattering $\rightarrow \mu_n$) through thermal fluctuations $k_B T$. This is an example of the <u>fluctuation - dissipation theorem</u>.

Another example might be drag, whose counterpart is Brownian motion. The Einstein relation would then be:

$$\underbrace{\frac{D}{\gamma}}_{\text{diffusivity / viscosity}} = \underbrace{k_B T}_{\text{thermal fluctuations}}$$

└ viscosity